

# An Example of the Use of STOZXFR

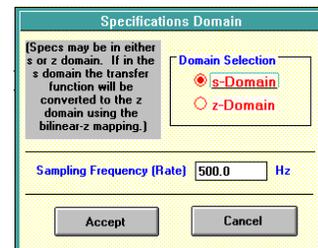
For this example we shall use an s-domain transfer function which can be considered to be the open-loop characteristic of a simple unity-feedback control system with an integrator, two lead-lag compensators, and a second-order plant. The details are

$$G(s) = \frac{775}{s} \cdot \frac{0.2s+1}{s+2} \cdot \frac{0.2s+1}{s+1} \cdot \frac{15267.1756}{s^2 + 100s + 15267.1756}$$

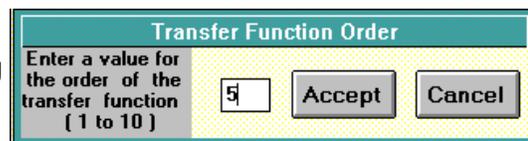
which we will enter into STOZXFR as a single transfer function in order to show the root-finding and factorization properties. The test transfer function will be

$$G(s) = \frac{4.7328244e5 s^2 + 7.0992366e6 s + 2.3664122e7}{s^5 + 103 s^4 + 15569.1756 s^3 + 46001.5268 s^2 + 30534.3512 s}$$

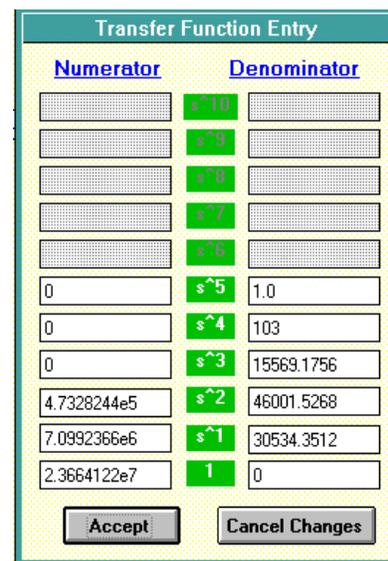
which will be entered by choosing the **File|TransferFunction|Keyboard Input** menu item. In the first dialog box we select the s-Domain option button and enter 500 as the sampling frequency. The choice of sampling frequency is important for two reasons: (1) the obvious one of avoiding significant aliasing of the analog signals which drive the digital control loop, and (2) the less obvious reason of avoiding a sampling frequency which is so high relative to the time constants in the transfer function that the resulting z-domain coefficients cannot be precisely represented, even in 32-bit floating point. (You can repeat this example with a sampling frequency of, say 5 kHz, to see that the low frequency response of the z-domain transfer function departs seriously from the analog prototype, principally due to coefficient values which are extremely close to the limits of 1.0 and ±2.0.)



In the next dialog box we choose the transfer function form for entering of the specifications. Then we enter the order of the transfer function in the next dialog box.



At this point the dialog box for the entry of the coefficients appears, with the text boxes for the higher orders of s disabled (grayed out). As is usual for Windows applications, movement from text box to text box is via the tab key. Be careful not to strike the key before the coefficients are all typed in; if you do you will probably see a message box stating that the transfer function is unstable. What goes on inside STOZXFR is that the roots of the numerator and denominator polynomials are computed, pairs of complex conjugate poles define biquad factors, and real poles are paired to form biquads and a single 1st-order section for an odd-order transfer function. Then the z-domain transfer function is computed using a proprietary version of the bilinear-z mapping. This proprietary method seeks to achieve as close a match as possible to the analog transfer function.



When the z-domain values have been computed the z-domain poles and zeros are in the Pole-Zero Plot and the numeric values are in the text boxes in the Pole/Zero Values frame. You cycle through the sections of the cascaded transfer function by clicking on the **Next Section** button. For an overall view, use the **ShowValues|Poles and Zeros** menu item to see all of the z-domain values.

z-Domain Pole and Zero Values						
----	Section #	Radius1	Angle(Hz)	Radius2	Type	↑
ZEROS	1	0.99005		0.980198	Double Real	
POLES		0.904835	17.9838		Complex Conj	
ZEROS	2	-1.0		-1.0	Double Real	
POLES		0.998002		0.996008	Double Real	
ZEROS	3	-1.0			Single Real	
POLES		1.0			Single Real	↓

OK

The **ShowValues|Coefficients** menu item displays the z-domain coefficients as

z-Domain Coefficients						
$b_0 z^2 + b_1 z + b_2$						
-----						
$a_0 z^2 + a_1 z + a_2$						
Sect # / Order	b0	b1	b2	a1	a2	↑
1 / 2nd (a0=1)	0.0755923	-0.148936	0.0733582	-1.76365	0.818726	
2 / 2nd (a0=1)	0.0755923	0.151185	0.0755923	-1.99401	0.994018	
3 / 1st (a0=0)		0.0755923	0.0755923	1.0	-1.0	↓

OK

To see the coefficients and poles of the s-domain transfer function (and also the z-domain), use the **ShowValues|Biquad Array** menu item (the BIQUAD array is an internal structure used to store the s-domain and z-domain specifications). Responding to the question in the input box by typing s will produce the first of these windows; the others appear when you click on the **Continue** button.

BIQUAD Array, s-Domain	
Section #	1
Pole Type	CC2
Zero Type	RL2
Pole Real	-50
Pole Imag	112.9919
Zero1	-10
Zero2	-5
a0	1
a1	100
a2	15267.18
b0	1
b1	15
b2	50
gain	77.93038

Continue

BIQUAD Array, s-Domain	
Section #	2
Pole Type	RL2
Zero Type	NUL
Pole1	-2
Pole2	-1
Zero1	
Zero2	
a0	1
a1	3
a2	2
b0	0
b1	0
b2	1
gain	77.93038

Continue

BIQUAD Array, s-Domain	
Section #	3
Pole Type	RL1
Zero Type	NUL
Pole1	0
Pole2	
Zero1	
Zero2	
a0	0
a1	1
a2	0
b0	0
b1	0
b2	1
gain	77.93038

Continue

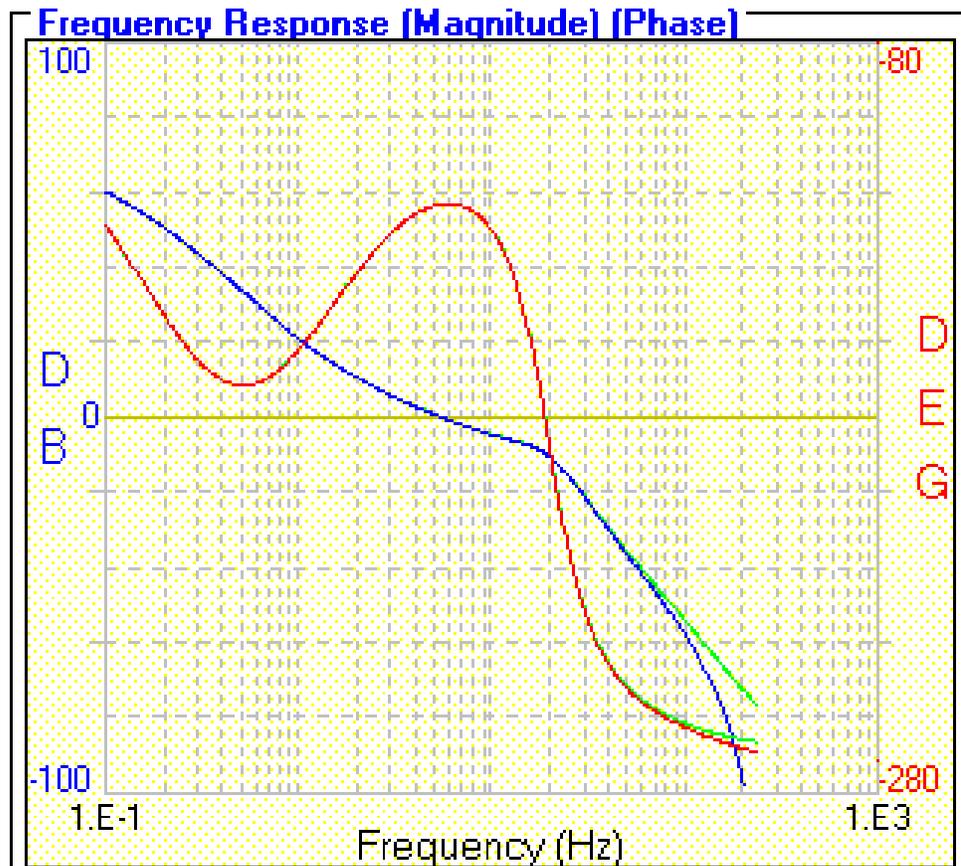
In this representation a gain value is displayed, corresponding to a b0 value of 1.0. If you wish to change the values in the s-domain transfer function you select the **File|TransferFunction|Edit|Transfer Function** menu item and the window used for the

original coefficient entry will appear. If you make some changes and then change you mind before clicking on the **Accept** button, you can void the changes by clicking on the **Cancel Changes** button.

For the frequency response computation and plotting the default choice is a linear frequency axis. To change to a logarithmic frequency axis just select the **Options|Freq Scale|Log** menu item (this puts the check mark next to **Log** instead of **Linear**). In the **Frequency Response Plot Parameters** frame there will be default parameters for the response computation and plotting. Decade values for the logF plot are specified as powers-of-ten. Click the **Compute** and then the **Plot** buttons to see the plot of magnitude and phase. The response of the s-domain prototype is plotted in green, and the z-domain responses are plotted on top of this s-domain response. Because the bilinear-z mapping can introduce zeros at one-half the sampling frequency, the z-domain response will roll off faster than the s-domain prototype as this folding frequency is approached. Usually, as in this example, the magnitude response is already down so far that it doesn't matter. For a more representative plot, under the assumption that the analog transfer function was for the open loop of a unity feedback system, change the plot parameters to a

-100 to 100 magnitude range, and a -280 to -80 degree phase range. Then click the **Clear + Replot** button to get this plot.

When numeric values are desired, you use the **Show Values|Frequency Response** menu item to display a table of the computed values in a scrollable list box. You can choose between the frequency response for the s-domain or the z-domain version.



If you need values at specific frequencies you should choose the **Linear** frequency scale and set the low and high frequency values together with the number of points so that you get the values and the resolution that you desire. The s-domain response values at the crossover

frequency neighborhood are

s-Domain FREQUENCY RESPONSE:			
Freq (Hz)	Magn (dB)	Phase (deg)	Delay (msec)
4.87528	0.991399	-123.812	-4.30148
4.96592	0.826285	-123.685	-3.87271
5.0	0.765267	-123.641	-3.58532
5.15229	0.499356	-123.467	-3.17238
5.24807	0.337538	-123.376	-2.64544
5.34564	0.17682	-123.297	-2.25505
5.44503	0.0172032	-123.23	-1.87292
5.54626	-0.141311	-123.175	-1.49916
5.64937	-0.298718	-123.133	-1.1335
5.7544	-0.455013	-123.104	-0.775235
5.86138	-0.610191	-123.088	-0.424124
5.97035	-0.764245	-123.084	-0.0803204
6.0	-0.805479	-123.086	0.134389
6.19441	-1.06895	-123.119	0.468962

**OK**

Treating this transfer function as representing an open-loop response, we can compare the values at the 0-dB and -180 degree points for the s-domain and z-domain responses. An approximate location of these points can be done using 1000 points over the -1 to +3 decades (0.1 Hz to 1 kHz) range. The result is

	Domain	Frequency (Hz)	Magnitude (dB)	Phase (Degrees)
0-dB Crossover Point	s	5.445	0.0172	-123.33
	z	5.445	0.0069	-123.288
Gain margin at 180 phase lag	s	18.8799	-9.47	-180.226
	z	18.8799	-9.594	-180.031

Clearly the z-domain equivalent is close enough to the s-domain prototype for engineering purposes.

The s-domain and z-domain coefficients can be saved to .TFF files (there is no external distinction between s and z coefficient files — you have to choose an appropriate name). Using the **File|TransferFunction|Save (.TFF)** menu item you have the choice between s- or z-domain parameters, and for z-domain coefficients the "original" or the modified values. This latter distinction applies only if you have "tweaked" the z-domain poles and/or zeros using the text boxes or the "spin buttons" in the **Pole/Zero Values** frame. We chose the files names EX\_S.TFF and EX\_Z.TFF. These files have the contents:

### EX\_S.TFF

```
<S> S-plane data
sampling frequency = 500
critical frequency = 0
number of sections = 3
order of sections = 2
<C> coefficients
77.93038,1168.956,3896.519,1,100,15267.18
0,0,77.93038,1,3,2
0,0,77.93038,0,1,0
```

### EX\_Z.TFF

```
<Z> Z-plane data--Fsamp = 500
number of sections = 3
order of sections = 2
<C> coefficients
7.559235E-02,-.1489357,.0733582,1,-1.763654,.8187256
7.559235E-02,.1511847,7.559235E-02,1,-1.99401,.994018
0,7.559235E-02,7.559235E-02,0,1,-1
```

With the coefficients stored in these files they may be reentered by choosing the menu item **File|TransferFunction|Open (.TFF or .FDF)**. Upon being read in the poles and zeros will be computed and displayed as before.

It should be clear that the alternative use of this STOZXFR application is to "play around" with the z-plane poles and zeros in order to get a desired frequency response. Because of the resolution of the full-scale plot it is difficult to adjust the response to within a few dB, unless you know that the pole or zero you are adjusting principally affects some small frequency region in which case you can "zoom" the frequency response plot to achieve greater visual resolution. In any case, playing with pole and zero values in the z-plane can be educational!

